1. Show that $\neg[\exists x A(x)]$ is equivalent to $\forall x[\neg A(x)]$.
   Answer: $\neg[\exists x A(x)]$ states that there does not exist an x that can satisfy $A(x)$. Therefore there is no x that will satisfy $A(x)$, and this can be expressed as $\forall x[\neg A(x)]$.

2. Prove that the following statement is false:

   There is an even prime bigger than 2

   Answer: A number is even if it can be divided by 2 with no remainder. If a number greater than 2 is even, then 2 is a factor of that number and it cannot be prime (according to the definition of prime numbers).

3. Translate the following sentences into symbolic form using quantifiers. In each case the assumed domain is given in parentheses.

   (a) All students like pizza. (All people)
   Answer: $\forall s \in P[likesPizza(s)]$

   (b) One of my friends does not have a car. (All people)
   Answer: $\exists! f \in P[friend(f) \wedge \neg ownsCar(f)]$

   (c) Some elephants do not like muffins. (All animals)
   Answer: $\exists e \in A[isElephant(e) \wedge \neg likesMuffins(e)]$

   (d) Every triangle is isosceles. (All geometric figures)
   Answer: Where t is a triangle, G is the set of all geometric figures and S is the set of all sides in a triangle, $\neg[\exists t \in G[\forall s \in S(\nexists s1 = s)]]$

   (e) Some of the students in the class are not here today. (All people)
   Answer: $\exists s \in P[student(s) \wedge \neg present(s)]$

   (f) Everyone loves somebody. (All people)
   Answer: $\forall p[\exists f \heartsuit p]$

   (g) Nobody loves everybody. (All people)
   Answer: $\neg[\exists p \in P[\forall f \heartsuit p]]$

   (h) If a man comes, all the women will leave. (All people)
   Answer: $\exists m \in P[comes(m) \Rightarrow leaves(\forall w)]$

   (i) All people are tall or short. (All people)
   Answer: $\forall p[short(p) \vee tall(p)]$

   (j) All people are tall or all people are short. (All people)
   Answer: $short(\forall p) \vee tall(\forall p)$

   (k) Not all precious stones are beautiful. (All stones)
   Answer: $\neg[\forall s \Rightarrow precious(s) \wedge beautiful(s)]$

   (l) Nobody loves me. (All people)
   Answer: $\neg[\exists p \heartsuit me]$

   (m) At least one American snake is poisonous. (All snakes)
   Answer: $\exists s[american(s) \wedge poisonous(s)]$

(n) At least one American snake is poisonous. (All animals)
Answer: $\exists s[snake(s) \wedge american(s) \wedge poisonous(s)]$

4. Negate each of the symbolic statements you wrote in the last question, putting your answers in positive form. Then express each negation in natural, idiomatic English.

(a) All students like pizza. (All people)
Answer: $\exists s \in P[\neg likesPizza(s)]$
There is a student who does not like pizza.

(b) One of my friends does not have a car. (All people)
Answer: $\forall f \in P[friend(f) \wedge ownsCar(f)]$
All of my friends have cars.

(c) Some elephants do not like muffins. (All animals)
Answer: $\forall e \in A[isElephant(e) \Rightarrow likesMuffins(e)]$
All elephants like muffins.

(d) Every triangle is isosceles. (All geometric figures)
Answer: $\exists t \in G[triangle(t) \wedge [\forall side1 \in t(\nexists side2 = s)]]$
There is a triangle that is not isosceles.

(e) Some of the students in the class are not here today. (All people)
Answer: $\forall s \in P[student(s) \Rightarrow present(s)]$
All students enrolled in the class are here today.

(f) Everyone loves somebody. (All people)
Answer: $\exists p \forall f\ \neg[p \heartsuit f]$
There is a person who doesn't love anybody.

(g) Nobody loves everybody. (All people)
Answer: $\exists p \forall f[p \heartsuit f]$
There is a person who loves everybody.

(h) If a man comes, all the women will leave. (All people)
Answer: $\forall m \forall w[isMan(m) \wedge comes(m) \not\Rightarrow woman(w) \wedge leave(w)]$
If any man comes, it is not the case that all the women will leave.

(i) All people are tall or short. (All people)
Answer: $\exists p[\neg tall(p) \wedge \neg short(p)]$
There is a person who is not tall or short.

(j) All people are tall or all people are short. (All people)
Answer: $\neg\forall p[short(p)] \vee \neg\forall p[tall(p)]$
It is not the case that all people are tall or all people are short.

(k) Not all precious stones are beautiful. (All stones)
Answer: $\forall s[precious(s) \Rightarrow beautiful(s)]$
All precious stones are beautiful.

(l) Nobody loves me. (All people)
Answer: $\exists p$ ♥ $me]$
There is a person who loves me.

(m) At least one American snake is poisonous. (All snakes)
Answer: $\nexists s[american(s) \wedge poisonous(s)]$
No American snakes are poisonous.

(n) At least one American snake is poisonous. (All animals)
Answer: $\nexists s[snake(s) \wedge american(s) \wedge poisonous(s)]$
No American snakes are poisonous.

5. Which of the following are true? The domain for each is given in parentheses.

(a) $\exists x(2x + 3 = 5x + 1)$ (Natural numbers)
Answer: False.

(b) $\exists x(x^2 = 2)$ (Rational numbers)
Answer: False.

(c) $\forall x \exists y(y = x^2)$ (Real numbers)
Answer: True.

(d) $\forall x \exists y(y = x^2)$ (Natural numbers)
Answer: False.

(e) $\forall x \exists y \forall z(xy = xz)$ (Real numbers)
Answer: False.

(f) $\forall x \exists y \forall z(xy = xz)$ (Prime numbers)
Answer: False.

(g) $\forall x[x<0 \Rightarrow \exists y(y^2 = x)]$ (Real numbers)
Answer: False.

(h) $\forall x[x<0 \Rightarrow \exists y(y^2 = x)]$ (Positive real numbers)
Answer: False.

6. Negate each of the statements in the last question, putting your answers in positive form.

(a) $\exists x(2x + 3 = 5x + 1)$ (Natural numbers)
Answer: $\forall x(2x + 3 \neq 5x + 1)$

(b) $\exists x(x^2 = 2)$ (Rational numbers)
Answer: $\forall x(x^2 \neq 2)$

(c) $\forall x \exists y(y = x^2)$ (Real numbers)
Answer: $\forall x(\nexists y = x^2)$

(d) $\forall x \exists y(y = x^2)$ (Natural numbers)
Answer: $\forall x(\nexists y = x^2)$

(e) $\forall x \exists y \forall z(xy = xz)$ (Real numbers)
Answer: $\forall x \forall y \forall z(xy \neq xz)$

(f) $\forall x \exists y \forall z (xy = xz)$ (Prime numbers)
Answer: $\forall x \forall y \forall z (xy \neq xz)$

(g) $\forall x [x<0 \Rightarrow \exists y (y^2 = x)]$ (Real numbers)
Answer: $\exists x [x<0 \ \wedge \ \nexists (y^2 = x)]$

(h) $\forall x [x<0 \Rightarrow \exists y (y^2 = x)]$ (Positive real numbers)
Answer: False.

7. Negate the following statements and put each answer into positive form:

   (a) $(\forall x \in N)(\exists y \in N)(x + y = 1)$
   Answer: $(\exists x \in N)(\nexists y \in N)(x + y = 1)$

   (b) $(\forall x{>}0)(\exists y{<}0)(x + y = 0)$ (where x, y are real number variables)
   Answer: $(\exists x{>}0)(\nexists y{<}0)(x + y = 0)$

   (c) $\exists x (\forall \epsilon{>}0)(-\epsilon{<}x{<}\epsilon)$ (where x, ϵ are real number variables)
   Answer: $\forall x (\nexists \epsilon{>}0)(-\epsilon{<}x{<}\epsilon)$

   (d) $(\forall x \in N)(\forall y \in N)(\exists z \in N)(x + y = z^2)$
   Answer: $(\exists x \in N)(\exists y \in N)(\nexists z \in N)(x + y = z^2)$

8. Give a negation (in positive form) of the famous "Abraham Lincoln sentence" which we met in the previous assignment: "You may fool all the people some of the time, you can even fool some of the people all of the time, but you cannot fool all of the people all the time."
   Answer: $(\exists t \exists p [\neg fooled(p)]) \wedge (\forall p \exists t [\neg fooled(p) \wedge \forall p \forall t [fooled(p)]$

9. The standard definition of a real function f being *continuous at a point* $x = a$ is

$$(\forall \epsilon{>}0)(\exists \delta{>}0)(\forall x)[|x - a|{<}\delta \Rightarrow |f(x) - f(a)|{<}\epsilon]$$

   Write down a formal definition for f being *discontinuous at a*. Your definition should be in positive form.